Int. J. Heat Mass Transfer. Vol. 5, pp. 763–764. Pergamon Press 1962. Printed in Great Britain.

RELATIONS FOR THE CRITICAL STATE DESCRIBING TRANSITION FROM LAMINAR TO TURBULENT FLOW IN FREE CONVECTION

L. S. KLYACHKO
All-Union Research Institute of Hydrotechnical and Sanitarytechnical Works, U.S.S.R.

(*Received* 12 *October* 1961)

Аннотация—Переход ламинарного режима переноса в турбулентный при свободной конвекции вдоль плоской поверхности и вокруг криволинейной характеризуется разным значением $ArPr$, равным в первом случае $ArPr \simeq 2{\cdot}8 \times 10^8$, а во втором $\simeq 2 \times 10^7$. Эти значения сохраняются для переноса тепла и массы.

NOMENCLATURE

Ar,	Arkhimed number, $gl^3/\nu^2 \cdot \Delta\rho/\rho$;
Pr,	Prandtl number, ν/d;
Sc,	Schmidt number, ν/k;
Nu,	Nusselt number, hl/x;
g,	gravitational constant, m/s^2;
l,	length (a diameter), m;
ν,	kinematic viscosity, m^2/s;
ρ,	density, kg s^2/m^4;
$\Delta\rho$,	change in density;
α,	thermal diffusivity, m^2/s;
λ,	thermal conductivity, kcal/h m degC;
h,	heat transfer, kcal/h m^2 degC;
k,	mass diffusivity, m^2/s.

Subscripts

cr,	critical;
con,	convection.

Superscripts

lam,	laminar;
tur,	turbulent.

In practical calculations and in experimental work on heat transfer, it is usual to assume that with free convection the flow regime is laminar in the region of $ArPr \leqslant 2 \times 10^7$ and turbulent at values of $ArPr > 2 \times 10^7$. No distinction is made for free convection round bodies and along plane surfaces. In all cases $ArPr$ is assumed to be equal to 2×10^7, i.e. this applies to volumetric and surface problems [1].

Experimental investigation [2] by the interference method of the structure of free flow along a vertical plane and observations of wave phenomena in a boundary layer of free flow and of the wavelength established that the beginning of transition of laminar flow regime into turbulent for the value $Pr \simeq 0{\cdot}71$ takes place at $Ar_{\text{cr}} \simeq 4 \times 10^8$, i.e. at

$$(ArPr)_{\text{cr}} \simeq 4 \times 10^8 \times 0{\cdot}71 = 2{\cdot}84 \times 10^8.$$

This value of $(ArPr)_{\text{cr}}$ is approximately fourteen times 2×10^7.

Naturally the question arises also whether the value $Ar_{\text{cr}} \simeq 4 \times 10^8$ is characteristic only for the plane problem, for which it has been experimentally determined, or it is general for volumetric problems as well.

In order to elucidate this question, consider the equality:

$$Nu^{\text{lam}}_{\text{free con}} = Nu^{\text{tur}}_{\text{free con}},$$

which in expanded form for a plane vertical surface may be written as:

$$0{\cdot}66(ArPr)^{1/4}_{\text{cr}} = 0{\cdot}135(ArPr)^{1/3}_{\text{cr}},$$

where 0·66 is taken from [3] and 0·135 from [1].

The solution of this equation gives:

$$(ArPr)^{1/12}_{\text{cr}} = 4{\cdot}9$$

or

$$(ArPr)_{\text{cr}} \simeq 2 \times 10^8.$$

At $Pr = 0{\cdot}71$

$$Ar_{\mathrm{cr}} = \frac{2 \times 10^8}{0{\cdot}71} \simeq 3 \times 10^8,$$

i.e. it has the order of the value

$$Ar_{\mathrm{cr\,exp}} \simeq 4 \times 10^8.$$

For complete correspondence it is sufficient that the numerical value of the coefficient 0·135 be substituted for 0·133. Then

$$Ar_{\mathrm{cr}} = Ar_{\mathrm{cr\,exp}} \simeq 4 \times 10^8.$$

Similarly proceeding from the equality

$$0{\cdot}54(ArPr)_{\mathrm{cr}}^{1/4} = 0{\cdot}133(ArPr)_{\mathrm{cr}}^{1/3},$$

we obtain for spherical bodies

$$(ArPr)_{\mathrm{cr}} \simeq 2 \times 10^7 \quad [1].$$

Thus it is true that numerical values of $(ArPr)_{\mathrm{cr}}$ differ for the two cases of dimensional and plane-surface convection.

Now let us show on the basis of joint consideration of energy-, heat- and mass-transfer equations that, for a laminar regime of free convection along a plane surface, the proportionality factor under the conditions of transfer is the same for all types of transfer and is equal to $\sim 0{\cdot}67$.

From the energy-transfer equations we have:

$$0{\cdot}667 Re_{\mathrm{cr}}^{0\cdot5} = 0{\cdot}037 Re_{\mathrm{cr}}^{0\cdot8}$$

and hence

$$Re_{\mathrm{cr}} \simeq 16\,200.$$

If we combine the turbulent heat transfer energy and heat equations in free convection:

$$0{\cdot}133(ArPr)_{\mathrm{cr}}^{1/3} = 0{\cdot}037 Re_{\mathrm{cr}}^{0\cdot8} = 0{\cdot}037(16\,200)^{0\cdot8},$$

we find that

$$(ArPr)_{\mathrm{cr}} \simeq 2{\cdot}25 \times 10^8.$$

This value of $(ArPr)_{\mathrm{cr}}$ is close to

$$(ArPr)_{\mathrm{cr}} = 2 \times 10^8$$

determined above.

Proceeding from the heat-transfer equation

$$A(ArPr)_{\mathrm{cr}}^{1/4} = 0{\cdot}133(ArPr)_{\mathrm{cr}}^{1/3},$$

we determine that

$$A = 0{\cdot}133(ArPr)_{\mathrm{cr}}^{1/12} = 0{\cdot}133(2{\cdot}25 \times 10^8)^{1/12} = 0{\cdot}66.$$

If we remember that experimental investigations [3] on evaporation of some liquids from a vertical surface at $ArSc \leqslant 3 \times 10^8$ yielded the value $A = 0{\cdot}66$ as well, it is possible to assume the existence of a fairly close analogy in transfer mechanism that allows calculation formulae to be written for heat- and mass-transfer processes on the basis of hydromechanical considerations.

REFERENCES

1. M. A. Mikheyev, *Fundamentals of Heat Transfer* (Osnovy teploperedachi). Gosenergoizdat, Moscow (1956).
2. E. R. G. Eckert, E. Söhngen and P. J. Schneider, *Transition of a Laminar Flow into Turbulent with Free Convection at a Vertical Plate* (in Russian translation). Sbornik "Problema pogranichnogo sloya i voprosy teploperedachi" pod redaktsiei V. A. Baum, Gosenergoizdat, Moscow (1960).
3. C. R. Wilke, C. W. Tolias and M. Eisenberg, Free-convection mass transfer at vertical plates. *Chem. Engng Progr.* **49**, 663–674 (1953).

Abstract—Transition of a laminar regime of transfer into the turbulent in free convection over a plane surface and round a curvilinear surface is represented by different values of *ArPr*, where in the first case $ArPr \simeq 2{\cdot}8 \times 10^8$ and in the second $\simeq 2 \times 10^7$. These values remain the same for both heat and mass transfer.

Résumé—La transition du régime laminaire au régime turbulent, en convection libre, sur une surface plane et sur une surface curviligne est représentée par des valeurs différentes de *ArPr*; dans le premier cas, on a $ArPr \simeq 2{,}8 \times 10^8$ et dans le second cas $\simeq 2 \times 10^7$. Ces valeurs restent les mêmes pour les transports de chaleur et de masse.

Zusammenfassung—Bei freier Konvektion an einer ebenen Platte und entlang einer krummlinigen Oberflächen ist der Übergang vom laminaren Übertragungsregim zum turbulenten durch bestimmte Werte von *ArPr* gekennzeichnet. Für den ersteren Fall ist $ArPr \simeq 2{,}8 \times 10^8$, für den zweiten Fall $\simeq 2 \times 10^7$. Diese Werte bleiben für Wärme- und Stoffübergang die gleichen.